A Wavelet Approach to the Verification of Probabilistic Spatial Fields

HD(CP)² High definition clouds and precipitation for advancing climate prediction

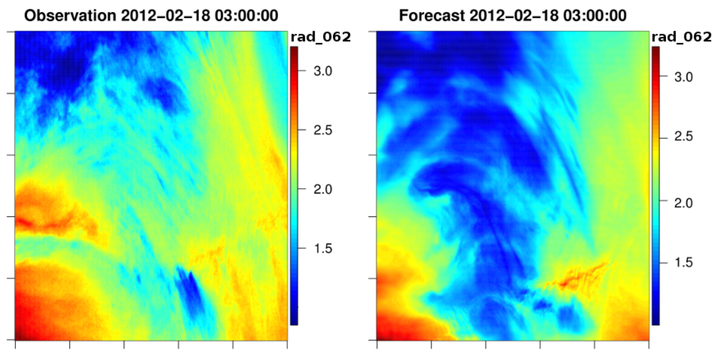
mweniger@uni-bonn.de

Michael Weniger, Petra Friederichs

Meteorological Institute, University Bonn

Aim

Evaluate **probabilistic** spatial fields (e.g. ensemble output) with non-negligible observational uncertainties.



Crucial Aspects

- **Robustness**: verification scores have to be robust w.r.t. observational uncertainties
- **Multivariate Probability Distributions**: in a probabilistic environment we do not perceive two spatial fields as fixed images, but rather as realizations of underlying multivariate probability distributions.
- **Curse of Dimensions**: even relatively small spatial fields lead to very high dimensional mathematical objects (e.g. covariance matrix).

Wavelet Approach

Haar *D4*

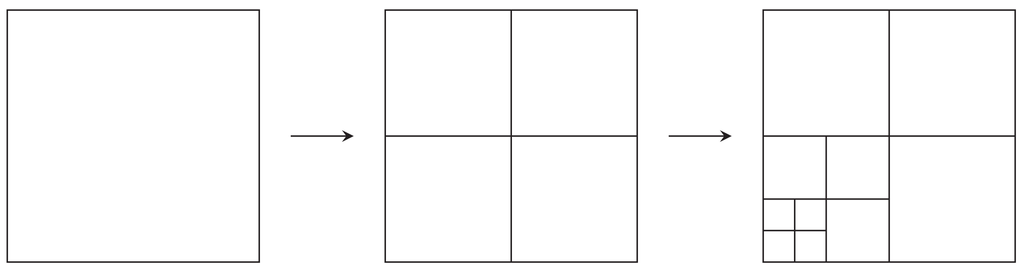
- Wavelets allow very efficient data reduction (e.g. JPEG image compression).
- There are reliable wavelet-based methods to handle data with significant amounts of noise. This is based on finding a "good" wavelet transform, i.e. wavelet basis functions, which emphasize the most important characteristics of a given set of data.

  ⇒ **best-basis-algorithms**
- **Wavelet Package Transforms (WPT)** extend these ideas to spatial fields. There are fast algorithms to find optimal wavelet-transformations based on minimizing user-defined cost functions (e.g. entropy of the energy-distribution of wavelet-coefficients).

  ⇒ The combination of efficient best-basis-algorithms with high data reduction allows to overcome the *curse of dimensions*.

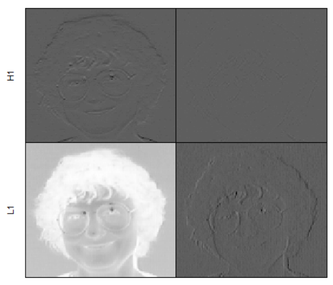
Two-Dimensional Wavelet Package Transform

Window-Scheme

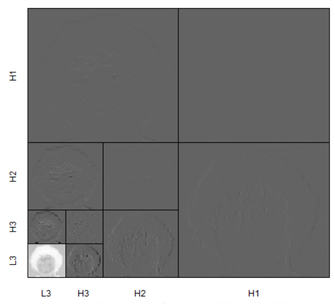


Example: Two-Dimensional D4-Wavelet Transform

Original Image



Level 1 WPT

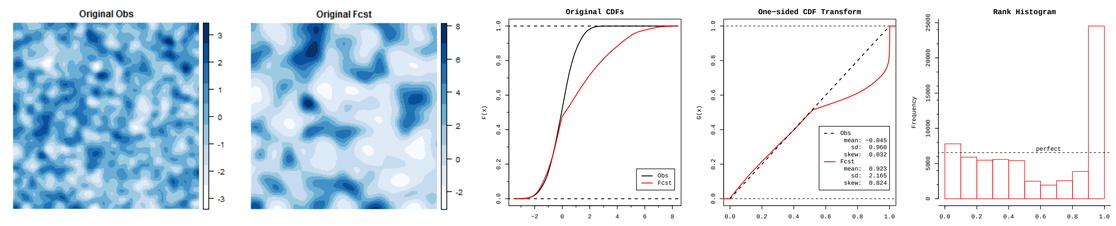


Level 3 WPT

Outline for a WPT Verification Method

1. Uni-variate intensity analysis
   ⇒ uni-variate normalization of spatial fields
2. Large-scale location errors → open question
3. Small-scale structure / texture analysis with WPT

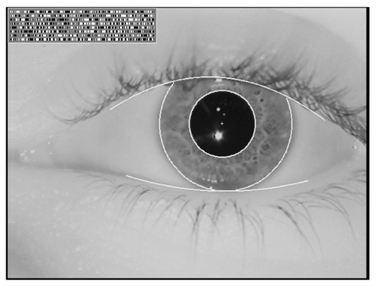
(1) Uni-Variate Intensity Analysis



- Forecast has an asymmetric bias (positive values are overestimated by a factor of 3)
- Uni-variate analysis separates intensity errors from scale-errors, i.e. ignores *texture*.
- CDFs can be analyzed using well-known one-dimensional statistical methods (e.g. Integrated Quadratic Distance, Earth-Movers Distance, Rank Histograms, etc.).

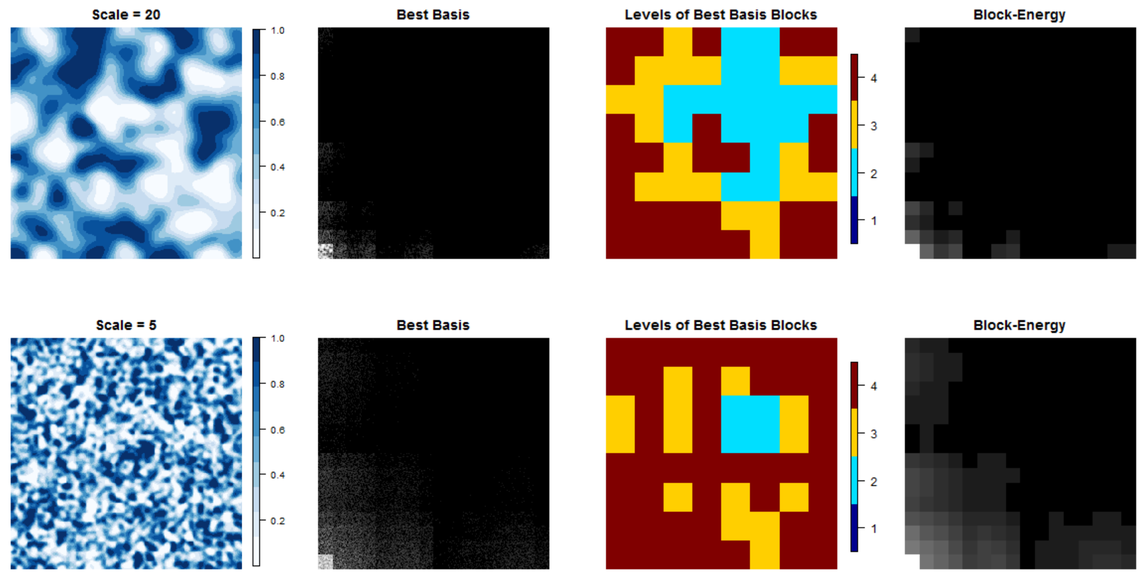
(3) Structure / Texture Analysis

Retina Scans [1]



- Structure of iris is decomposed with 2D-Gabor wavelets
- Matching-algorithm uses information measures on wavelet coefficients.

Texture-Analysis on Normalized Fields



Open Questions

- Exact mathematical definition of metrics for
  - ▷ uni-variate intensity analysis
  - ▷ index-vectors of best-wavelet-bases
  - ▷ energy distributions of wavelet coefficients
- Definition / method to describe displacement errors, e.g.
  - ▷ Image Warping
  - ▷ de-noised center of mass evaluation

References

[1] John Daugman. New methods in iris recognition. *Systems, Man, and Cybernetics, Part B: Cybernetics, IEEE Transactions on*, 37(5):1167–1175, 2007.