Image Warping for Forecast Verification

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Methodology

Motivation

Transform (deform) the forecast field, F, to look as much like the observed field, O, as possible.

Information about forecast performance given by:

- Traditional score(s), $\boldsymbol{\theta}$, of un-deformed field, F, against O.
- Percent reduction in $\boldsymbol{\theta}$ after Affine deformations (η_1).
- Percent reduction in $\boldsymbol{\theta}_{\text{affine}}$ after Nonlinear deformations (η_2).
- Amount of movement necessary to improve $\boldsymbol{\theta}$ by η_1.
- Amount of bending energy required to further improve $\boldsymbol{\theta}_{\text{affine}}$ by η_2.

Methodology

$$\tilde{F}(\boldsymbol{s}) = F(W(\boldsymbol{s})),\ \boldsymbol{s} \in \mathcal{D}$$

where $\mathcal{D}$ is the support of the image (i.e., the grid).

$$W(\boldsymbol{s}) = W_{\mathrm{NL}}(W_{\mathrm{affine}}(\boldsymbol{s}))$$

maps coordinates from the undeformed image, F, to the deformed image, $\tilde{F}$.

Methodology

Many choices for W. A few popular choices.

- polynomials (e.g., Alexander et al., 1999; Dickinson and Brown, 1996)
- thin-plate splines (e.g., Sampson and Guttorp, 1992; Glasbey and Mardia, 2001; Åberg et al., 2005).
- B-splines (e.g., Lindström, Gilleland and Lindgren (In Prep))
- Other (e.g. Keil and Craig, 2007)

Motivation

For computational concerns, use control points, $\boldsymbol{p}^F$ and $\boldsymbol{p}^O$, to determine the warp.

Introduce *log*-likelihood to measure dissimilarity between $\tilde{F}$ and O. This is different from measuring via a forecast verification score!

$$\log \mathrm{p}(O|F, \boldsymbol{p}^F, \boldsymbol{p}^O) = h(\tilde{F}, O) \tag{1}$$

where choice of error likelihood h depends on the forecast variable.

Methodology

Must penalize non-physical warps!

Introduce a smoothness prior for the warps

Behavior determined by the control points. Assume these points are fixed and apriori *known*, in order to reduce the prior on the warping function to $\mathrm{p}(\boldsymbol{p}^F|\boldsymbol{p}^O)$.

$$\mathrm{p}(\boldsymbol{p}^F|O, F, \boldsymbol{p}^O) = \log \mathrm{p}(O|F, \boldsymbol{p}^F, \boldsymbol{p}^O)\mathrm{p}(\boldsymbol{p}^F|\boldsymbol{p}^O) \tag{2}$$

where it is assumed that $\boldsymbol{p}^F$ are conditionally independent of F given $\boldsymbol{p}^O$.

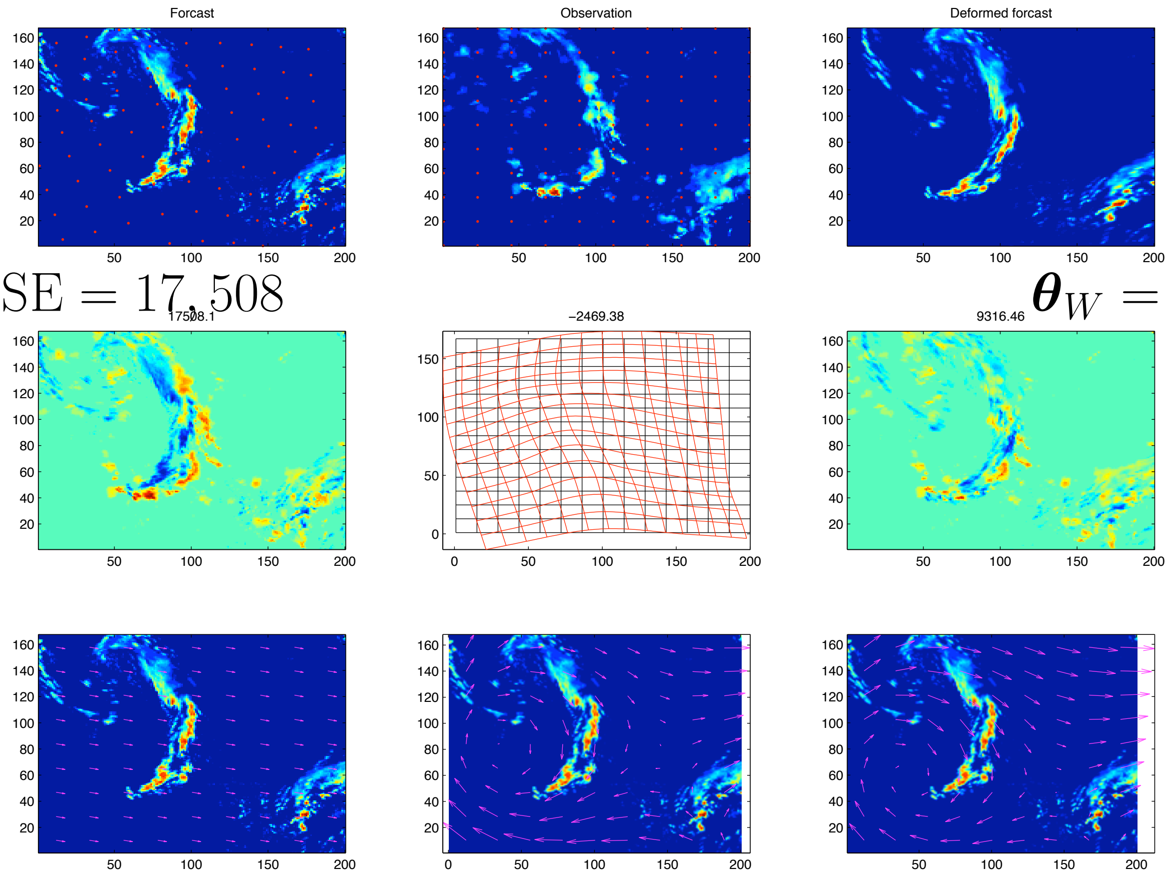
Methodology

Estimation

To find the optimal deformation (based on $\boldsymbol{p}^F$ and $\boldsymbol{p}^O$), maximize the likelihood given by (2). From (1) and (2), we get

$$\begin{aligned}\ell(\boldsymbol{p}^F|O,F,\boldsymbol{p}^O) &= \log \mathrm{p}(O|F,\boldsymbol{p}^F,\boldsymbol{p}^O) + \log \mathrm{p}(\boldsymbol{p}^F|\boldsymbol{p}^O) \\ &= h(\tilde{F},O) + \log \mathrm{p}(\boldsymbol{p}^F|\boldsymbol{p}^O).\end{aligned}$$

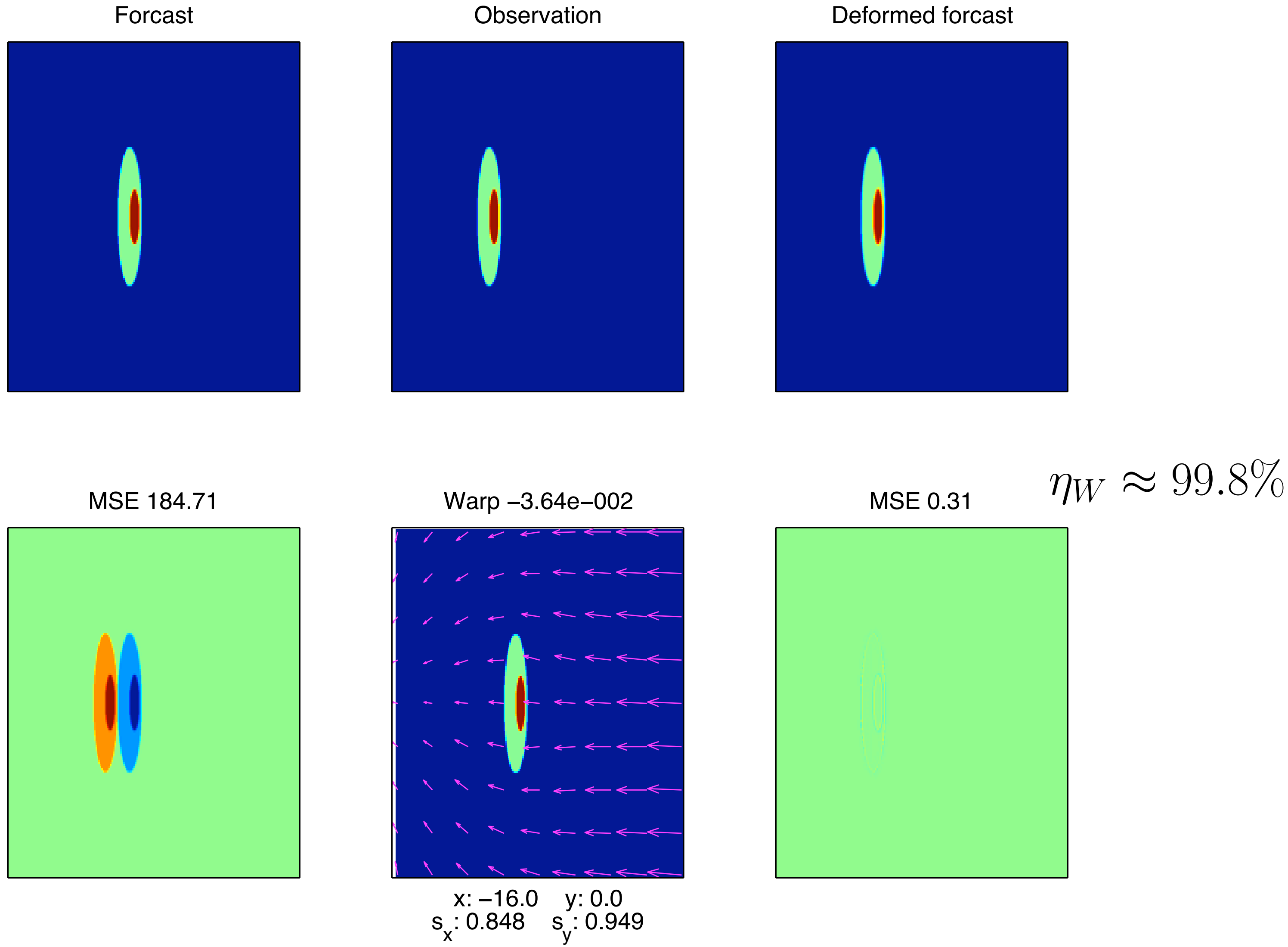
Test Cases



$\boldsymbol{\theta} = \text{MSE} = 17,508$

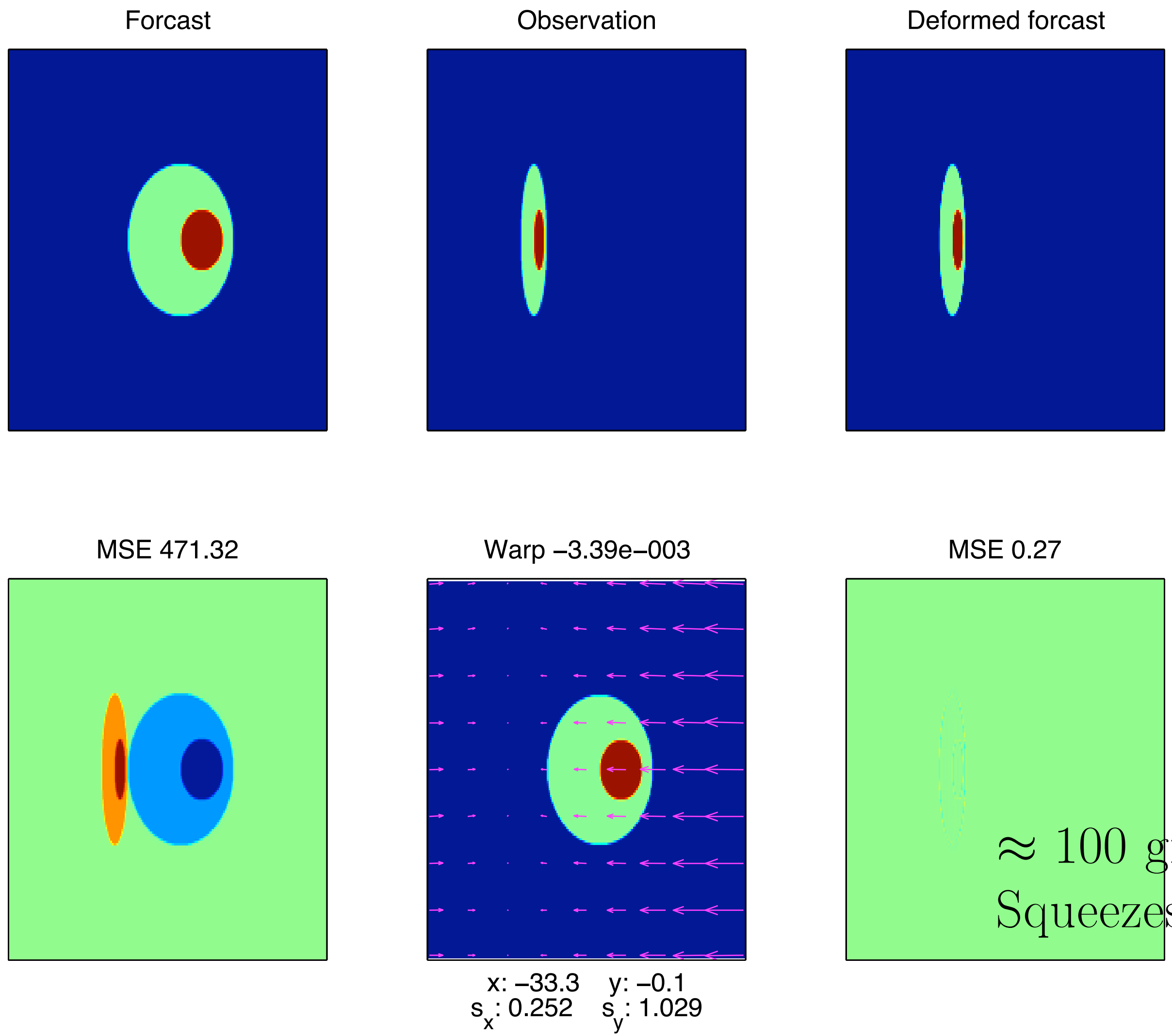
$\boldsymbol{\theta}_W = 9316$

$$\eta_W = \frac{17{,}508 - 9316}{17{,}508} \approx 47\%$$



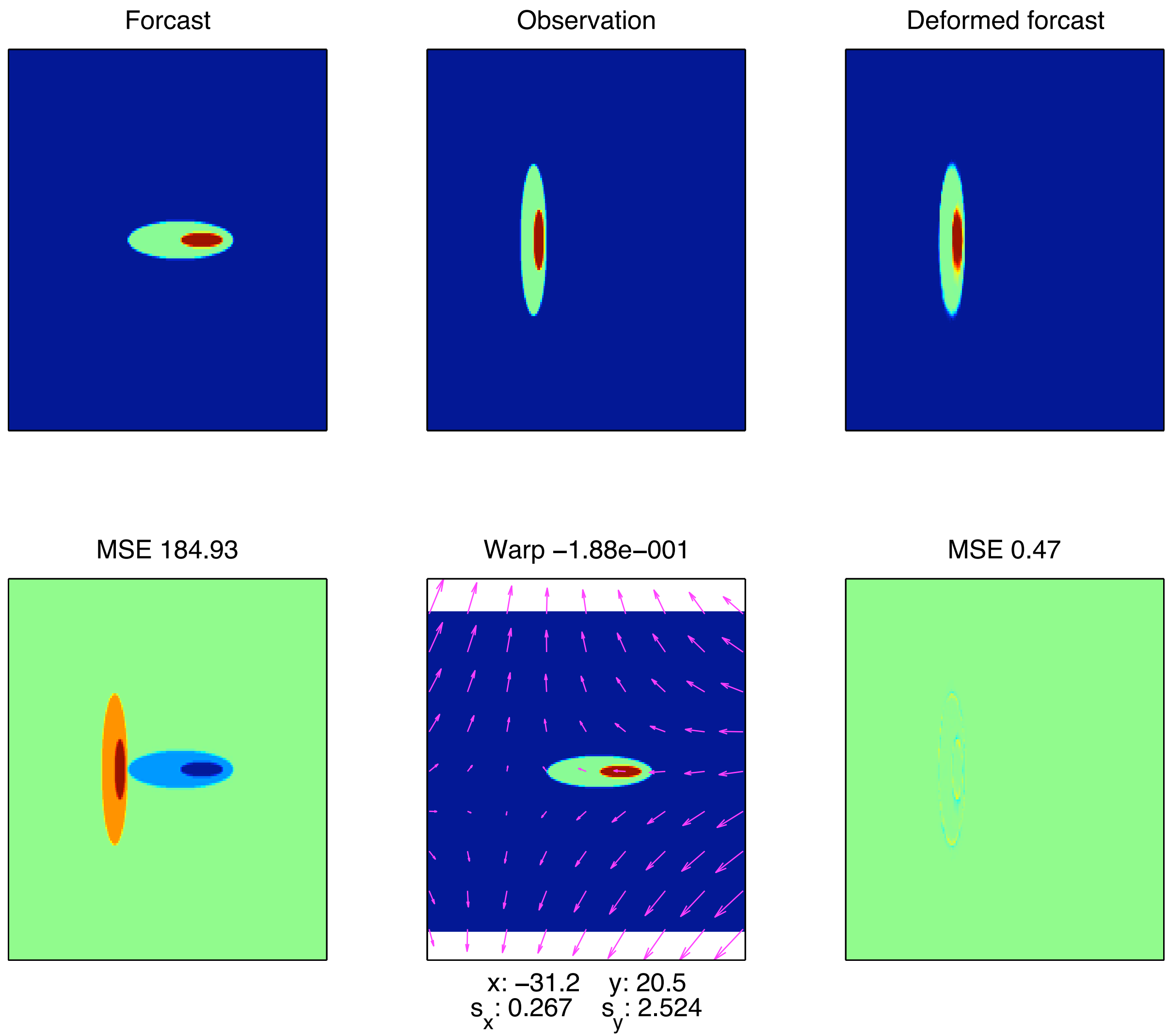
Gemoetric 1; 50 pts too far to the east

$3 \cdot (-16.0) = -48 \equiv$ Moves forecast 48 grid points to the west; negligible re-scaling and nonlinear movement.

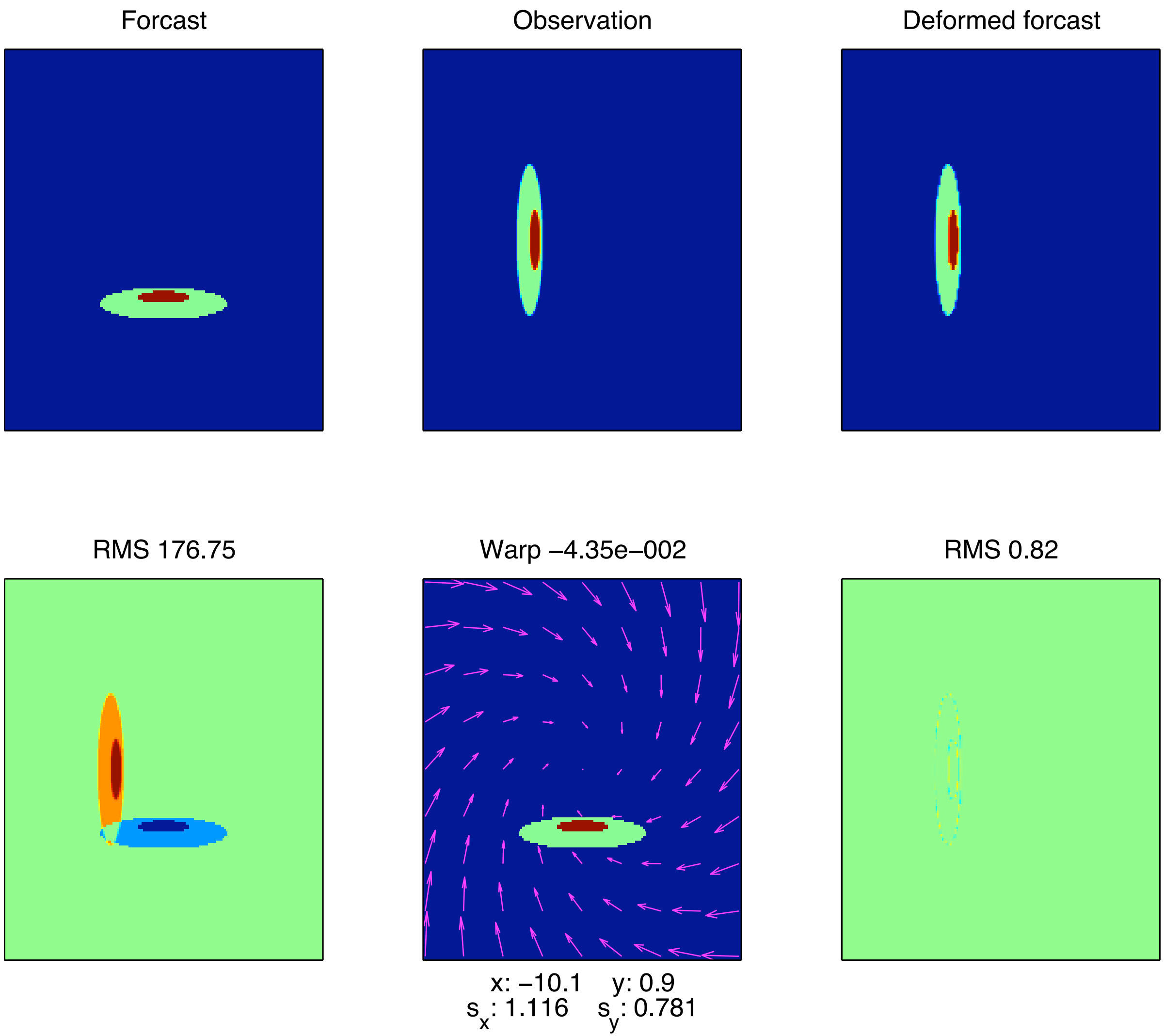


≈ 100 grid points west
Squeezes horizontally.

Geometric 3; 125 grid points too far east and larger spatial coverage



Geometric 4; 125 pts too far east and incorrect orientation



True Rotation

Discussion, Ongoing and Future Work

- Rotations vs. Re-scaling is tricky!
- Control points (fewer mean faster computation, but less intricate warps).
- Statistical model will allow for confidence intervals.
- Works well on binary images as well as real cases.
- Potentially can be used on most any field (e.g., wind vector fields, temperature, etc.)
- Extendable to multiple dimensions (time, vertical, etc.)
- Gives information about types of error.
- thin-plate splines move pixels globally, better to use B-splines?

References

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